Journal of Statistical Physics, Vol. 7, No. 1, 1973

Letter to the Editor

Comment on the Paper "Jaynes's Maximum Entropy Prescription and Probability Theory"[1]

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Received April 27, 1972

In a recent note [*J. Stat. Phys.* 3(4):381 (1971)] Friedman and Shimony claim to have found a case in which assignment of probabilities by Jaynes's maximum-entropy prescription is inconsistent with general principles of probability theory. This claim is too serious to let pass without comment, since it would controvert the universal applicability of the max-entropy algorithm.

The mathematical argument of Friedman and Shimony is correct. But we find it to be merely an interesting way of deriving in a special circumstance something we already know to be true in general. They are led to the opposite conclusion by certain inaccuracies in interpretation. These inaccuracies are introduced in two key sentences which we discuss here.

[1] The paper "Jaynes's Maximum Entropy Prescription and Probability Theory," by Kenneth Friedman and Abner Shimony, appeared in *J. Stat. Phys.* 3(4):381 (1971).

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First, just before their Eq. (4) they introduce $\hat{d}_\epsilon$ as the "evidence that the posterior expected value of (a dynamical variable) E is ϵ." Later they admit that this definition of $\hat{d}_\epsilon$ is less than clear. However, their use of $\hat{d}_\epsilon$ in connection with Jaynes's algorithm *demands* that it is equivalent to the well-defined proposition "the expected value of E is ϵ."

Second, just following their Eq. (6) Friedman and Shimony state that "the background information b does not in general imply a definite value of ϵ." On the contrary, the background information includes well-defined propositions which determine a definite value for the expected value of E. Indeed, Friedman and Shimony assume that b specifies the existence of a system which can be in one and only one of n distinct states. On the basis of this assumption alone they rightly conclude by max-entropy inference that

$$P(h_i \mid b) = 1/n$$

[their Eq. (3)], where h_i is the proposition that the system is in the ith state. Now, b must also include the proposition that "the dynamical variable E has the value E_i when the system is in the ith state"; otherwise E is not sufficiently well-defined even to talk about. Then (3) gives

$$\epsilon = \sum_{i=1}^{n} E_i P(h_i \mid b) = (1/n) \sum_{i=1}^{n} E_i$$

for the expected value ϵ of E. Thus, by max-entropy inference, b implies a unique value for ϵ. This can be expressed alternatively by writing

$$P(\hat{d}_\epsilon \mid b) = \delta\left(\epsilon - \frac{1}{n}\sum_i E_i\right)$$

where $P(\hat{d}_\epsilon \mid b)$ is the "probability (density) that, given b, the expected value of E is ϵ." But this expression for $P(\hat{d}_\epsilon \mid b)$ is identical to the equation (9) which Friedman and Shimony show by a mathematical argument to hold in the special case that $(1/n) \sum_i E_i = E_m$ for some state m.

They misinterpret their result as "the inferrability with certainty from b that evidence will be forthcoming which will fix the posterior expected value of E to be E_m, the same as the prior expected value" and claim that it is inconsistent with a probability assignment which "honestly describes what we know." On the contrary, as we have explained, their result is already required by general considerations, and so presents no conceptual difficulties to Jaynes's approach.